Control Synthesis for Flexible Space Structures Excited by Persistent Disturbances

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Both classical and state-space synthesis methods for active control of flexible space structures in the presence of persistent disturbances are presented. The methods exploit the so-called internal model principle for asymptotic disturbance rejection. A generic example of flexible space structures is used to illustrate the simplicity of the proposed design methodologies. The concept of a disturbance rejection dipole is introduced from a classical control viewpoint. It is shown that the proposed design methods will invariably make use of disturbance rejection dipoles and non-minimum-phase compensation for a class of noncolocated control problems in the presence of persistent disturbances. The need for tradeoffs between performance and parameter robustness is also discussed.

I. Introduction

THIS paper presents active vibration control synthesis methodologies that exploit the internal model principle for asymptotic disturbance rejection. The robust servomechanism problem for asymptotic tracking and disturbance rejection has been investigated by many researchers over the last three decades, and it seems that the internal model principle is the most intuitive solution to such a control problem. A detailed discussion and an extensive list of references for this subject can be found in Ref. 1. It is interesting to note that use of the frequency-shaped, linear quadratic Gaussian (LQG) technique developed in Ref. 2 results in a disturbance rejection controller that has the disturbance model poles. This is basically the essence of the internal model principle.

In this paper, both classical and state-space synthesis methods for the active control of flexible space structures in the presence of persistent disturbances are developed. Although the proposed methods utilize many well-known concepts and methodologies, some significant contributions of the paper are the following: 1) it is shown that the standard state-space control design techniques, such as pole-placement and LQG control techniques, can be simply employed for the active suppression of persistent disturbances; 2) the concept of a disturbance rejection dipole is introduced from a classical control viewpoint; and 3) a generic example of flexible space structures[3-5] is used to illustrate the simplicity of the proposed design methodologies. It is emphasized that the proposed approaches are different from other techniques that are based on disturbance estimation. The simplicity of the dipole concept was experimentally demonstrated for the Mini-Mast truss structure[6] and the ACES testbed[7] through the NASA Control Structure Interaction Guest Investigator Program.

In Ref. 8, a multivariable disturbance rejection control design employing the internal model principle was first proposed for the Space Station, and the importance of the transmission zero concept in asymptotic disturbance rejection was demonstrated. In Ref. 9, a robust H_∞ control synthesis technique for structured parameter uncertainty was applied to the same control design problem of Ref. 8, resulting in a significant improvement in parameter robustness margins. In Ref. 10, a robust H_∞ control technique was also developed for robust asymptotic disturbance rejection in the presence of structured parameter uncertainty as well as persistent excitation and applied to the same example problem of this paper. This paper, however, emphasizes the simplicity of the standard classical and state-space design approaches over the H_∞ control design methodology of Ref. 10.

This paper is organized as follows. In Sec. II, a classical frequency-domain synthesis technique is presented for flexible space structures with persistent excitation. This simple classical approach is applied to a two-mass-spring generic model of flexible space structures. In Sec. III, two state-space methods, the LQG synthesis technique and the pole-placement technique, are employed to find suitable compensation for active disturbance rejection. The concept of internal modeling of a persistent disturbance is again exploited, introducing a state-space model of the disturbance into the feedback loop. The state-space design methods are then applied to the two-mass-spring example. Disturbance rejection is achieved in each case by introducing into the feedback loop an internal model of the disturbance.

II. Classical Frequency-Domain Design

The design of a single-input/single-output (SISO) feedback control system for a free-free flexible structure is carried out starting with the stabilization of the rigid-body mode and subsequent analysis and stabilization of unstably interacting flexible modes.[3,4] Feedback control with a noncollocated actuator and sensor pair generally results in the presence of unstably interacting flexible modes. After the unstably interacting modes have been identified, proper filtering to phase or gain stabilize those modes is then introduced. Also, active disturbance rejection filtering is synthesized to compensate for any persistent disturbances acting on the structure. Aided by the root locus method and/or Bode plots and a certain amount of trial and error, a robust compensator design is obtained. The classical SISO design based on successive mode stabilization can be divided into four steps: 1) rigid-body mode stabilization according to given time or frequency domain specifications (settling time, maximum overshoot, bandwidth, phase and gain margins); 2) stabilization or active control of any

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unstably interacting flexible modes; 3) synthesis of active disturbance rejection filters; and 4) final tuning of the overall compensator. This last step becomes necessary because the phase and gain characteristics of active disturbance rejection filtering as well as the stabilized modes in question exert their influence on all neighboring frequencies, which may include other modes. This presents a challenge as the number of modes to be stabilized becomes larger.

Rigid-Body Mode Control

Control of the rigid-body mode is simply achieved with proportional plus derivative (PD) type feedback. For practical purposes, however, a phase-lead filter or PD control with a second-order rolloff filter is used. The rigid-body mode should have reasonable damping for a satisfactory settling time since the overall transient response is often dominated by the rigid-body mode.

Flexible-Mode Stabilization

Phase and/or gain stabilization of an unstably interacting flexible mode can be achieved with the introduction of a rolloff filter and/or a generalized second-order filter of the following form in the feedback loop:

$$\frac{s^2/\omega_z^2 + 2\zeta_z s/\omega_z + 1}{s^2/\omega_p^2 + 2\zeta_p s/\omega_p + 1}$$

where s is the Laplace transform variable. Non-minimum-phase second-order shaping filters with negative ζ_z are of special interest for a certain class of noncollocated control problems, as discovered in Ref. 4.

Phase stabilization of a flexible mode can be achieved by introducing certain phase lead or lag in the feedback loop as needed so that the phase margin corresponding to that mode becomes positive, thus rendering the unstably interacting mode stable for a given loop gain. The use of second-order non-minimum-phase filters, which have negative ζ_z, is very suitable for this purpose since the range of phase lag extends from 0 to -360 deg as opposed to the maximum of about 90 deg of phase lead or lag that any practical, second-order minimum-phase filter provides. The goal is to introduce phase lag at the frequency corresponding to a particular, unstably interacting mode, such that the real parts of the closed-loop eigenvalues corresponding to that mode become negative and as large as possible, while minimizing the gain increase in the high-frequency range. The exact gain and phase characteristics of these second-order shaping filters is determined by the location of its poles and zeros. The non-minimum-phase shaping filter introduces phase lag ranging from 0 to -360 deg over a certain frequency band. The filter can be synthesized such that a desired phase lag occurs at a particular frequency. The simplest filter is the non-minimum-phase, all-pass filter, where $\omega_z = \omega_p$ and $\zeta_z = -\zeta_p < 0$. This filter has a -180-deg phase lag at ω_p and no gain change at all frequencies, which makes it very convenient for phase stabilization. The damping ratios of the filter poles and zeros influence the sensitivity of the stabilized mode to variations in the modal frequencies due to plant parameter uncertainties. A greater damping makes the stable mode more robust. On the other hand, greater damping increases the influence of the filter on the rest of the modal frequencies, which may be undesirable.

Gain stabilization of an unstably interacting flexible mode can be achieved only if that mode has a certain amount of passive damping. If the damping at that mode is satisfactory, it can be gain stabilized by attenuating the gain at that particular mode so that it remains stable for a given loop gain. The larger the passive damping at a particular mode, the more conveniently it can be gain stabilized. Usually, gain stabilization is applied in order to stabilize higher modes that have no significant effects on the overall performance. In practice, a structure has a certain amount of passive damping, which allows for the convenient gain stabilization of such modes. Gain stabilization is generally performed with the use of first- or second-order rolloff filters, or notch filters.

Active Disturbance Rejection

After successful stabilization of the rigid-body mode as well as any other unstably interacting flexible modes, active disturbance rejection is then simply achieved by introducing into the feedback loop a model of the disturbance. It is assumed that a persistent disturbance is represented as

$$w(t) = \sum_i A_i \sin(p_i t + \phi_i)$$

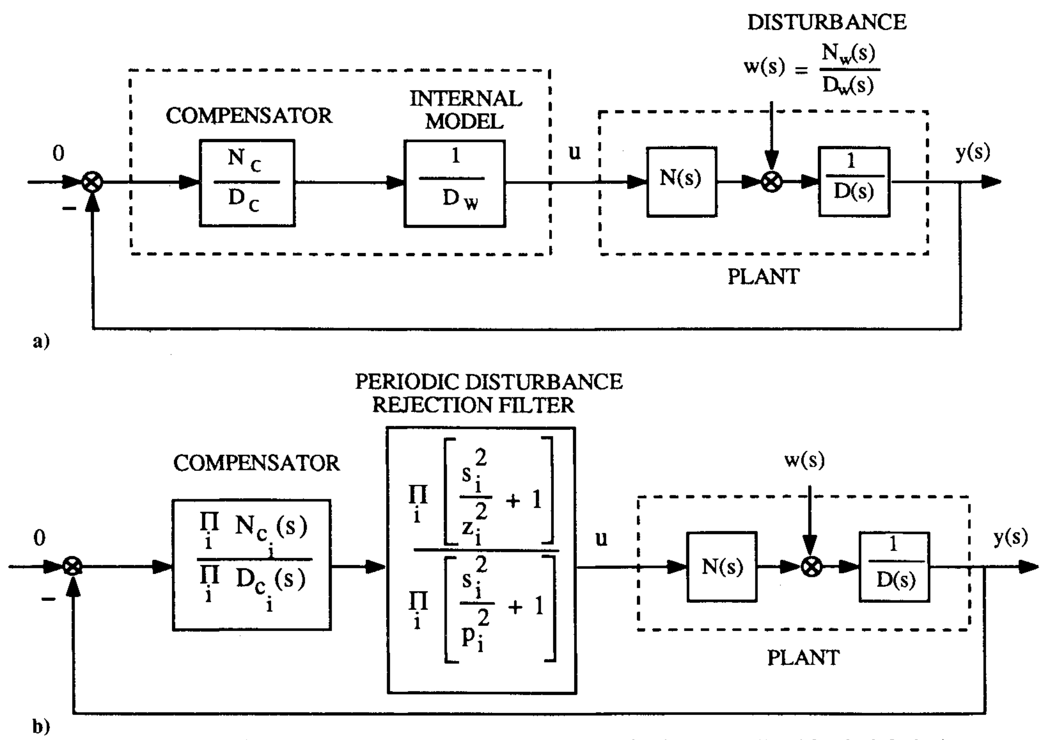

Fig. 1 Block diagram representation of a disturbance rejection controller (classical design).

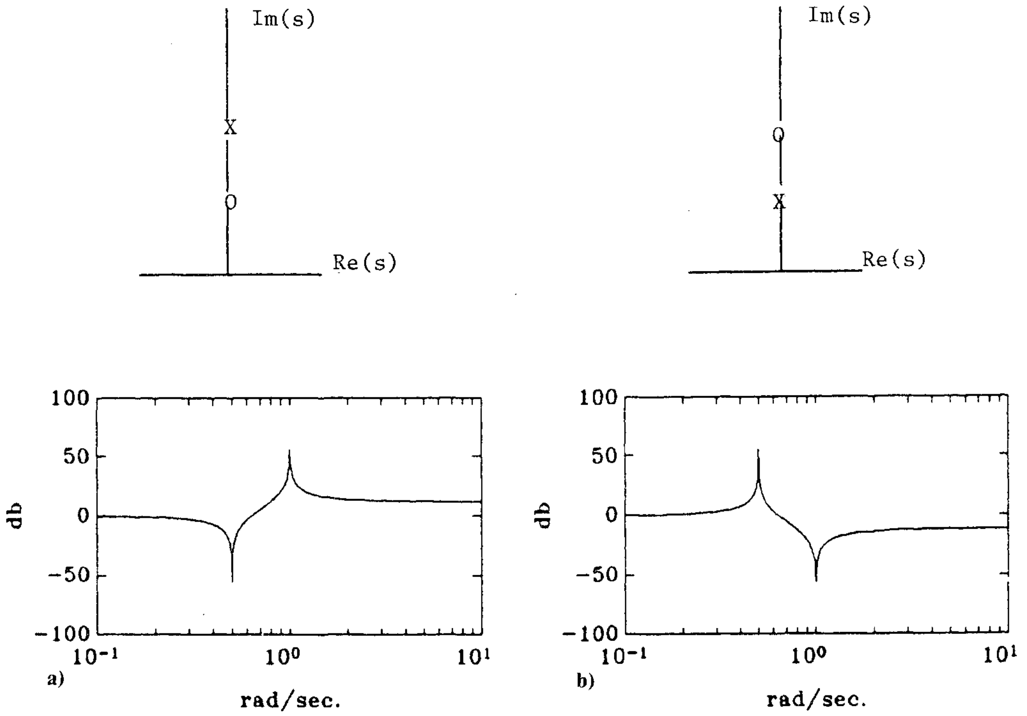

Fig. 2 Disturbance rejection dipoles: a) $\omega_p = 1$, $\omega_z = 0.5$; b) $\omega_p = 0.5$, $\omega_z = 1$.

with unknown magnitudes A_i and phases ϕ_i but known frequencies p_i. In general, the disturbance $w(t)$ can be described by a Laplace transformation $w(s) = N_w(s)/D_w(s)$, where $N_w(s)$ is arbitrary as long as $w(s)$ remains proper. The roots of $D_w(s)$ correspond to the frequencies at which the persistent excitation takes place. The inclusion of the disturbance model $1/D_w$ inside the control loop is often referred to as the internal modeling of the disturbance.[1] In classical design, the internal disturbance model is regarded as being part of the compensator, as shown in Fig. 1a. The presence of $1/D_w$ in the control loop results in the effective cancellation of the poles of $w(s)$, provided that no root of $D_w(s)$ is a zero of the plant transfer function. This is shown in the following closed-loop transfer function:

$$\begin{aligned} y(s) &= \frac{1/D(s)}{1 + N_c(s)N(s)/D_c(s)D_w(s)D(s)} w(s) \\ &= \frac{D_c(s)D_w(s)}{D_w(s)D_c(s)D(s) + N_c(s)N(s)} \frac{N_w(s)}{D_w(s)} \end{aligned} \tag{1}$$

where we can see the cancellation of $D_w(s)$.

The compensator can be viewed as a series of individual first-order or second-order filters as follows,

$$\frac{N_c(s)}{D_c(s)} = \prod_i \frac{N_{c_i}(s)}{D_{c_i}(s)}$$

Each filter is designed to perform a specific task, like the stabilization of a particular mode. In the same manner, a disturbance rejection filter can be designed that has a proper transfer function and uses the internal disturbance model $1/D_w$ so that Eqs. (1) remain true. Thus, a proper numerator is chosen in the compensator to go with the disturbance model, as shown in Fig. 1b. The numerator is chosen to be of the same order as D_w so that there is a zero for each pole of the disturbance model $1/D_w$.

Although the asymptotic disturbance rejection control concept based on the internal model principle has been well known (e.g., see Ref. 1), a new interpretation of the concept from a classical control viewpoint is presented here. Each pole-zero combination of the disturbance rejection filter

$$\prod_i \frac{s^2/\omega_{z_i}^2 + 2\zeta_{z_i} s/\omega_{z_i} + 1}{s^2/\omega_{p_i}^2 + 1}$$

can be called a dipole, where ζ_{z_i} is included for generality. The filter thus consists of as many dipoles as there are frequency components in the persistent disturbance. The separation between the zero and the pole is generally referred to as the strength of the dipole. The strength of the dipole affects the settling time of the closed-loop system; in general, the larger the separation between the pole and zero of the filter, the shorter the settling time is. This is caused by the position of the closed-loop eigenvalue corresponding to the filter dipole. As the strength of the dipole is increased, this eigenvalue is pushed farther to the left, speeding up the response time of the disturbance rejection. However, this separation influences the gain/phase characteristics of the system since the dipole presents a certain amount of gain/phase changes in its neighborhood. Moreover, at frequencies higher than the dipole, there is a net gain increase or reduction, as shown in Figs. 2 for a case with $\zeta_z = 0$. The magnitude of this gain increases with the separation between pole and zero. Therefore, as the strength of the dipole is changed to meet a chosen settling time, the compensation must be readjusted. A compromise has to be reached often between the settling time and the stability of the compensated system.

Example

Consider a two-mass-spring system shown in Fig. 3, which is a simple generic example of flexible space structures.[3-5] It is assumed that the two bodies both have nominal mass $m_1 = m_2 = 1$ and are connected by a spring with nominal stiffness $k = 1$. A control force u acts on body 1, and the position of body 2 is measured as y, resulting in a noncollocated control problem. Assume that a sinusoidal disturbance $w(t) = \sin(0.5t)$ with unknown magnitude and phase is exerted on body 1 and/or body 2 and that asymptotic disturbance rejection for body 2 with a settling time of 20 s is to be achieved.

The dynamical equations of this system are

$$m_1\ddot{x}_1 + k(x_1 - x_2) = u + w_1 \tag{2a}$$

$$m_2\ddot{x}_1 + k(x_2 - x_1) = w_2 \tag{2b}$$

where x_1 and x_2 are the displacements of bodies 1 and 2, respectively. For the nominal system, the modal equations become

$$\ddot{q}_1 = 0.5(u + w_1 + w_2) \tag{3a}$$

$$\ddot{q}_2 + 2q_2 = 0.5(u + w_1 + w_2) \tag{3b}$$

where $q_1 = (x_1 + x_2)/2$ and $q_2 = (x_1 - x_2)/2$, and the transfer function from $u(s)$ to $y(s)$ also becomes

$$\frac{y(s)}{u(s)} = \frac{1}{s^2(s^2+2)}$$

For a settling time of 20 s and reasonable closed-loop damping, the following compensator without a disturbance rejection filter component is first chosen,

$$u(s) = -0.16\frac{[(s/0.2)+1][(s/2.01)^2 - 2(0.5)(s/2.01)+1]}{[(s/2)+1][(s/2.01)^2+2(0.5)(s/2.01)+1]}y(s)$$

The inclusion of a disturbance rejection filter, however, may significantly alter the closed-loop characteristics of the preceding design. The amount of phase/gain changes caused by the addition of a disturbance rejection filter dipole is proportional to the strength of the dipole, as discussed earlier. The disturbance filter poles need to be placed at $\pm 0.5j$. If the corresponding zeros are placed at $\pm 0.499j$, the strength of the dipole is minimal and the system is altered very little, but although the disturbance is canceled eventually, the settling time is unacceptable. Increasing the separation between the poles and zeros reduces the settling time of the system. If the zeros are placed at $\pm 0.4j$, however, the dipole shifts the phase of the system at the frequency of the stable rigid-body mode so that the rigid-body mode becomes less stable and readjustment of all of the compensator parameters becomes necessary. Figure 4 shows a root locus vs overall gain for the readjusted closed-loop system with the following compensator:

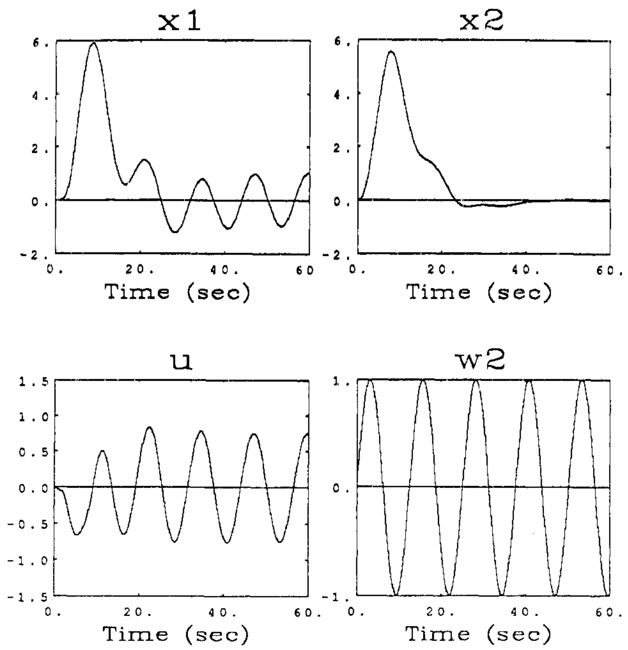

Fig. 5 Time responses to $w_2(t) = \sin(0.5t)$ for classical design.

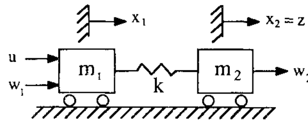

Fig. 3 Two-mass-spring example.

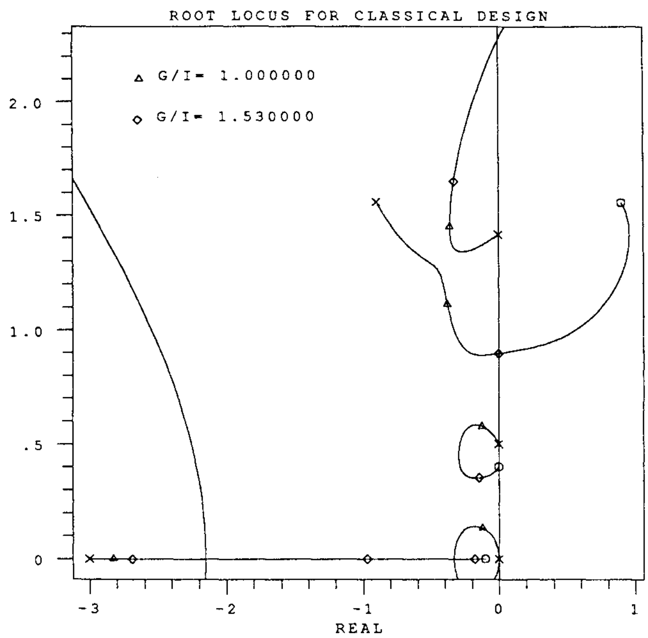

Fig. 4 Root locus vs overall gain for classical design.

$$u(s) = -0.04\frac{[(s/0.1)+1][(s/1.8)^2 - 2(0.5)(s/1.8)+1]}{[(s/3)+1][(s/1.8)^2+2(0.5)(s/1.8)+1]}$$

$$\cdot\frac{[(s/0.4)^2+1]}{[(s/0.5)^2+1]}y(s)$$

As can be seen in Fig. 4 (where $G/I = 1.0$ and 1.53 indicate the overall gain of 1 and 1.53, respectively), the closed-loop system has a 3.7 dB gain margin. The closed-loop system is stable for the following spring stiffness range:

$$0.66 \leq k \leq 2.32$$

It is important to note that the dipole's zeros cannot be arbitrarily placed around the dipole's poles. The zeros have to be properly placed, in general, between consecutive poles of the system, in this case the plant poles at the origin and the dipole's poles. Otherwise, the artificial flexible mode, introduced by the disturbance rejection filter dipole, becomes unstable. The time responses to $w_2(t) = \sin(0.5t)$ for the nominal system are shown in Fig. 5. It can be seen that, at steady state, body 1 has to oscillate at the disturbance frequency for the control of body 2.

To overcome some drawbacks of the classical successive-mode-stabilization design approach, a more systematic approach using state-space synthesis techniques is developed in the next section.

III. State-Space Control Design

It is assumed that a plant is described by the state-space equation

$$\dot{x}_p = A_p x_p + B_p u + G_p w \tag{4a}$$

$$y = C_p x_p + v \tag{4b}$$

where x_p is the plant's state vector, u the control input, w the process noise, and v the measurement noise. Both w and v are assumed to be white noise processes with

$$E[w(t)w^T(\tau)] = W\delta(t-\tau)$$

$$E[v(t)v^T(\tau)] = V\delta(t-\tau)$$

where W and V are the corresponding spectral densities.

In general, a compensator for this plant will consist of a regulator and an estimator that will approximate the states x_p with estimated states $\hat{x}_p$ using the information from the measured output y. The estimator attempts to asymptotically reduce the error term $e = \hat{x}_p - x_p$. A controller consisting of an estimator and an estimated state feedback controller is then given by

$$\begin{aligned}\dot{\hat{x}}_p &= A_p\hat{x}_p + B_pu + L(y - C_p\hat{x}_p)\\ &= (A_p - LC_p)\hat{x}_p + B_pu + Ly \end{aligned} \tag{5a}$$

$$u = -k\hat{x}_p \tag{5b}$$

where the term $y - C_p\hat{x}_p$ represents the error between the output of the plant and the estimated output, and K and L are gain matrices to be determined.

The internal model principle is incorporated with the standard control design problem discussed earlier. Active disturbance rejection for the measured output is achieved by introducing a model of the disturbance inside the control loop, therefore using again the concept of internal modeling. Consider a persistent disturbance with one or more frequency components represented as

$$w(t) = \sum_i A_i \sin(p_i t + \phi_i)$$

with unknown magnitudes A_i and phases ϕ_i but known frequencies p_i. A disturbance rejection filter for $w_i(t)$ at a particular frequency is then modeled as

$$\dot{x}_{wi} = A_{wi}x_{wi} + B_{wi}y \tag{6}$$

where

$$A_{wi} = \begin{bmatrix} 0 & 1 \\ -p_i^2 & 0 \end{bmatrix}, \qquad B_{wi} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The internal model then includes as many frequencies as the given disturbance and is driven by the measured output y of the plant. This procedure is equivalent to the one used in the classical approach with the disturbance model now consisting of a state-space model. The disturbance rejection filter is then described by

$$\dot{x}_d = A_dx_d + B_dy \tag{7}$$

where x_d is the metastate vector introduced by the internal disturbance model, A_d is block-diagonal and contains A_{wi}, for each disturbance $w_i(t)$ and B_d also contains B_{wi} for each disturbance.

The disturbance filter model described by Eq. (7) is then augmented to the plant described by Eqs. (4) as follows,

$$\dot{x} = Ax + Bu + Gw \tag{8a}$$

$$y = Cx + v \tag{8b}$$

where

$$x = \begin{bmatrix} x_p \\ x_d \end{bmatrix}, \quad A = \begin{bmatrix} A_p & 0 \\ B_dC_p & A_d \end{bmatrix}, \quad B = \begin{bmatrix} B_p \\ 0 \end{bmatrix}$$

$$C = [C_p \quad 0], \quad G = \begin{bmatrix} G_p \\ 0 \end{bmatrix}$$

An estimated state feedback controller is then given as

$$u = -K\hat{x}$$

where $\hat{x} = [\hat{x}_p^T \quad \hat{x}_d^T]$, and the gain matrix K is to be determined for the augmented system described by Eqs. (8). As shown in Fig. 6, the metastate x_d is, however, directly fed back as

$$u = -[K_p \quad K_d]\begin{bmatrix} \hat{x}_p \\ x_d \end{bmatrix} \tag{9}$$

since x_d is directly available from Eq. (7).

An active disturbance rejection controller in state-space form is then given by

$$\begin{bmatrix} \dot{\hat{x}}_p \\ \dot{x}_d \end{bmatrix} = \begin{bmatrix} A_p - B_pK_p - LC_p & -B_pK_d \\ 0 & A_d \end{bmatrix}\begin{bmatrix} \hat{x}_p \\ x_d \end{bmatrix} + \begin{bmatrix} L \\ B_d \end{bmatrix} y \tag{10a}$$

$$u = -[K_p \quad K_d]\begin{bmatrix} \hat{x}_p \\ x_d \end{bmatrix} \tag{10b}$$

The closed-loop system with $w = v = 0$ is then described by

$$\begin{bmatrix} \dot{x}_p \\ \dot{\hat{x}}_p \\ \dot{x}_d \end{bmatrix} = \begin{bmatrix} A_p & -B_pK_p & -B_pK_d \\ LC_p & A_p - B_pK_p - LC_p & -B_pK_d \\ B_dC_p & 0 & A_d \end{bmatrix}\begin{bmatrix} x_p \\ \hat{x}_p \\ x_d \end{bmatrix}$$

This equation can be modified using the error term $e = \hat{x}_p - x_p$, resulting in a partially decoupled system of equations as follows,

$$\begin{bmatrix} \dot{x}_p \\ \dot{x}_d \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A_p - B_pK_p & -B_pK_d & -B_dK_p \\ B_dC_p & A_d & 0 \\ 0 & 0 & A_p - LC_p \end{bmatrix}\begin{bmatrix} x_p \\ x_d \\ e \end{bmatrix}$$

The determinant of this matrix is equal to the determinants of the diagonal submatrices multiplied together, one giving the regulator eigenvalues for the augmented system including the internal model and the other giving the estimator eigenvalues for only the plant. Hence, the separation principle for regulator and estimator[11] holds for a system even with an internal model for asymptotic disturbance rejection.

Linear Quadratic Gaussian Control Synthesis

In this section, the standard LQG control synthesis technique[11] is employed for the selection of controller gain matrices $K = [K_p \quad K_d]$ and L in Eqs. (10). The LQG technique is briefly summarized here with some necessary changes, though simple, to accommodate the internal model principle.

The optimal control $u = -Kx$ is first considered by minimizing the quadratic performance index

$$J = \frac{1}{2}\int_0^\infty (x^TQx + u^TRu)\, dt \tag{11}$$

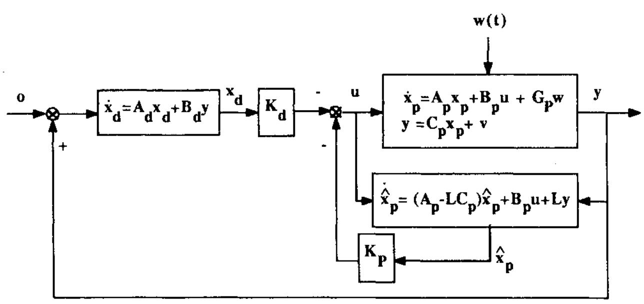

Fig. 6 Block diagram representation of a disturbance rejection controller (state-space design).

where $x = [x_p^T \quad x_d^T]$ is the state vector of the augmented system, Q the state weighting matrix, and R the control input weighting matrix.

The regulator gain matrix K is then obtained as

$$K = R^{-1}B^TX \tag{12}$$

by solving the Riccati equation

$$0 = XA + A^TX - XBR^{-1}B^TX + Q \tag{13}$$

where A is the augmented system matrix in Eq. (8a).

The estimator gain matrix L is selected such that the observation error

$$e = \hat{x}_p(t) - x_p(t)$$

$$u(s) = \frac{-0.355[(s/0.2375)+1][(s/0.496)^2 + 2(0.204)(s/0.496)+1]}{[(s/2.545)^2 + 2(0.204)(s/2.545)+1][(s/0.5)^2+1]} \cdot \frac{[(s/1.41)^2 - 2(0.0546)(s/1.41)+1]}{[(s/2.873)^2 + 2(0.859)(s/2.873)+1]} y(s) \tag{16}$$

is minimized in the presence of noise, by solving the Riccati equation

$$0 = A_pY + YA_p^T - YC_p^TV^{-1}C_pY + G_pWG_p^T \tag{14}$$

where A_p is the unaugmented plant system matrix, Y the estimate error covariance matrix, and L is found as

$$L = YC_p^TV^{-1} \tag{15}$$

It is emphasized that the use of the frequency-shaped LQG methodology[2] also results in solving these two Riccati equations. Before applying the LQG approach to the example problem of this paper, we briefly summarize the pole-placement technique and then apply these two state-space approaches to the example problem.

Pole-Placement Technique

The pole placement technique basically allows the designer to directly choose the closed-loop regulator and estimator eigenvalues to meet desired criteria. For a single input system, the Bass-Gura method[12] computes the state feedback gain matrix K as

$$K = (\hat{a} - a)(\mathfrak{J}^{-T}\mathfrak{C})^{-1}$$

where $\mathfrak{J}$ is a lower triangular Toeplitz matrix with first column $[1, a]^T$, $\mathfrak{C}$ the controllability matrix for (A, B), and a and $\hat{a}$ the row vectors containing the coefficients of the characteristic polynomials for A (the augmented system matrix) and $\hat{A}$ (an associated system matrix having the desired eigenvalues), respectively.

The estimator gain matrix L is computed using duality, the selection of the closed-loop eigenvalues for the estimator being independent from that of the regulator. Thus, the Bass-Gura method can then be rephrased by letting

$$A \to A_p^T, \qquad B \to C_p^T, \qquad K \to L^T$$

resulting in the Bass-Gura formula for L given by

$$L = (\mathfrak{O}\,\mathfrak{J})^{-T}(\breve{a} - a)^T$$

where $\mathfrak{O}$ is the observability matrix for (A_p, C_p), and a and $\breve{a}$ are row vectors containing the coefficients of the characteristic polynomials for A_p (the unaugmented system matrix) and $\breve{A}_p$ (an associated system matrix having the desired eigenvalues), respectively.

Although the closed-loop eigenvalues can be arbitrarily chosen, not all selections result in good designs. Although the settling time of the compensated system depends on the real part of the regulator closed-loop eigenvalues, the real part of these eigenvalues cannot be arbitrarily large since a faster decay means a larger input signal. Also, this influence increases with the frequency of the eigenvalue, so that faster decay of high-frequency modes means even more control input effort.

Example

Consider the example problem discussed in Sec. II to illustrate the state-space approaches for asymptotic disturbance rejection control design. First, the LQG approach is applied to the problem, followed by the pole-placement approach.

After trial and error, an LQG controller can be found as

It can be seen that for asymptotic disturbance rejection, the compensator has poles at $\pm 0.5j$ with the associated zeros near $\pm 0.5j$. Such a pole-zero pair is called a disturbance rejection filter dipole.

Figure 7 shows a closed-loop root locus vs overall gain of this LQG controller. The closed-loop system has a 1.8-dB gain margin and is stable for only $0.9 \leq k \leq 1.16$ ($-0.1 \leq \Delta k \leq 0.16$). The standard LQG control design is necessarily tuned closely to the plant model for high performance; hence, it is not robust to plant parameter uncertainty. The responses to $w_2(t) = \sin(0.5t)$ for the nominal system are shown in Fig. 8. It can be seen that the transient peak is, however, very small compared to the responses (Fig. 5) of the previous classical control design with non-minimum-phase zeros. It is clear that for the LQG design high performance (small transient peak and fast settling time) has been achieved at the expense of a small stability robustness margin with respect to parameter uncertainty. Hence, some tradeoffs between performance and parameter robustness must be considered in practical control design. However, LQG-type controllers can also be designed for parameter robustness, which has been one of the ongoing research topics in robust control.[10]

Consider the pole-placement approach. The regulator gain matrix K is computed for the augmented system, including the

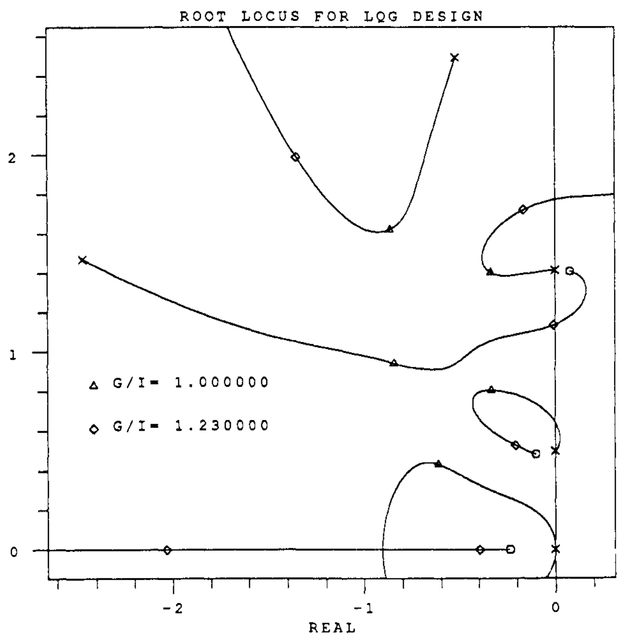

Fig. 7 Root locus vs overall gain for LQG design.

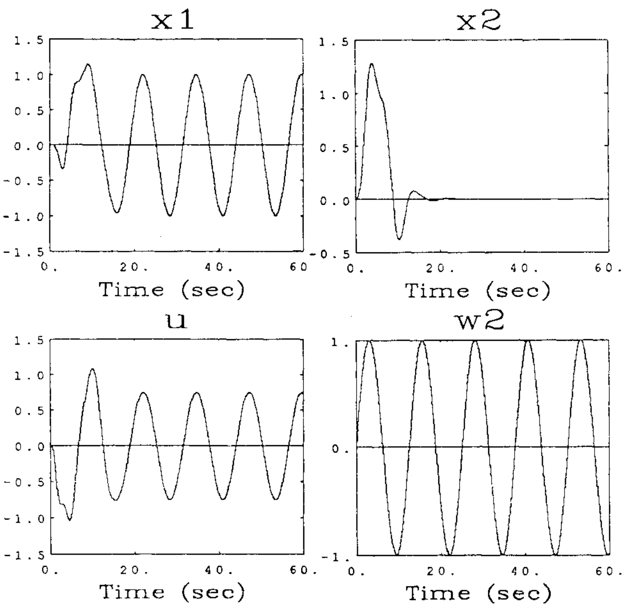

Fig. 8 Time responses to $w_2(t) = \sin(0.5t)$ for LQG design.

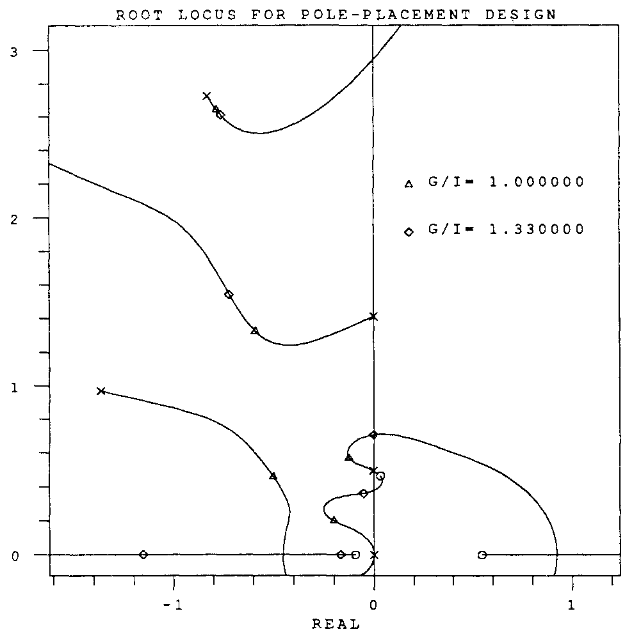

Fig. 9 Root locus vs overall gain for pole-placement design.

internal model, for given desired closed-loop eigenvalues. The estimator gain matrix L is computed using only the plant system matrices since only the plant states are to be estimated. The regulator eigenvalues are tentatively chosen to be similar to the closed-loop eigenvalues resulting from the classical design as follows:

The estimator eigenvalues (only for the plant) are then chosen to be twice as far as the regulator eigenvalues. The resulting compensator is

$$u(s) = \frac{-0.0354[(s/0.0942)+1][-(s/0.544)+1][(s/4.617)+1]}{[(s/0.5)^2+1][(s/1.672)^2+2(0.815)(s/1.672)+1]} \cdot \frac{[(s/0.467)^2-2(0.073)(s/0.467)+1]}{[(s/2.849)^2+2(0.29)(s/2.849)+1]} y(s) \qquad (17)$$

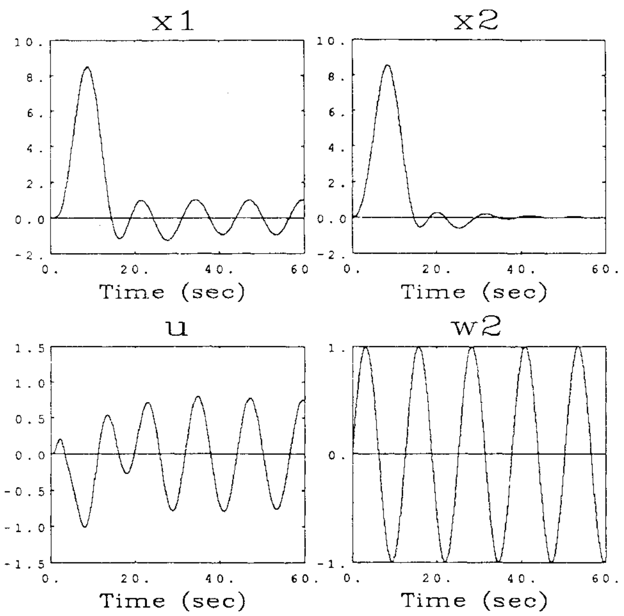

Fig. 10 Time responses to $w_2(t) = \sin(0.5t)$ for pole-placement design.

It can be seen that for asymptotic disturbance rejection the compensator has poles at $\pm 0.5j$ with the associated zeros near them.

Figure 9 shows a closed-loop root locus vs overall gain of this controller. It can be seen that the rigid-body mode is stabilized by a PD-type compensator with a second-order rolloff filter and the flexible mode is stabilized with a nonminimum-phase filter. Also, note that the pole-placement algorithm also introduces a dipole for disturbance rejection. It also introduces a real, positive zero to go with the complex pair of poles for the stabilization of the unstably interacting flexible mode. The placement of the eigenvalue corresponding to the disturbance rejection filter determines the location of this zero. The real part of the eigenvalue influences the settling time while the imaginary part affects the magnitude of the response. The smaller the imaginary component of the eigenvalue, the larger the magnitude of the overshoot in the response of the system as well as in the control input signal. The settling time determined by the real component of the eigenvalue is not altered by the overshoot.

The closed-loop system is stable for $0.57 \leq k \leq 3.55$ and has a 2.48-dB gain margin. The responses to $w_2(t) = \sin(0.5t)$ are shown in Fig. 10. Similar to the classical design, this controller designed using the pole-placement technique has a parameter robustness margin bigger than that of the LQG design, but it has a very large transient peak. It is again evident that some tradeoffs between performance and robustness are needed even for this simple example problem.

IV. Discussion and Summary

State-space methods for control design of flexible space structures are currently emphasized and more widely explored than classical methods. This arises from the convenience of obtaining a compensator for the whole system given one set of design parameters (e.g., given Q, R, W, and V, or desired closed-loop eigenvalues), as demonstrated in the previous sections. In classical design, on the other hand, a compensator must be constructed piece by piece, or mode by mode. However, both classical and state-space methods have their drawbacks as well as advantages.

As shown in the previous sections, both state-space techniques (pole placement and LQG) introduce non-minimum-phase filtering of the unstably interacting flexible modes. The LQG technique offers an optimal compensator design in the presence of random disturbances given certain weighting parameters for the states and the control inputs and certain parameters describing the random disturbances. The question remains how to choose these parameters and what choice provides the best optimal design. The designer must find an acceptable set of parameters for a good optimal design. The use of state-space methods for control design usually results in a compensator of the same order as the system to be controlled. This means that, for systems having several flexible modes, the compensator adds compensation even to modes that are stable and need no compensation. This may result in a complicated compensator design.

The classical design is particularly convenient for the control of flexible space structures with well-separated flexible modes. The concept of non-minimum-phase compensation also provides an extremely convenient way of stabilizing unstably interacting flexible modes, as discovered in Ref. 4. The resulting compensator is usually of less order than the system to be controlled since not all flexible modes in a structure tend to be destabilized by a reduced-order controller. A helpful characteristic of most flexible space structures is their inherent passive damping. This gives the designer the opportunity of phase stabilizing significant modes and of gain stabilizing all other higher frequency modes that have less influence on the structure. On the other hand, successive mode stabilization presents problems of its own, and a retuning of the compensated system becomes necessary. It is also noticed that reducing the damping in a frequency shaping filter reduces its influence on neighboring frequencies and it also reduces the phase lag at lower frequencies. However, reducing the damping of the filters increases the sensitivity of the phase stabilized modes to plant parameter uncertainties.

Active disturbance rejection is achieved in both the classical methods and state-space methods, with the introduction of an internal model of the disturbance into the feedback loop. The concept of internal modeling of the disturbance works as well with a classical transfer function description as with a state-space description. In the classical design, the internal modeling of the disturbance leads to the introduction of a disturbance rejection dipole, or filter, for each frequency component of the disturbance. In the state-space design, the introduction of the internal model results in the addition of two metastates for each frequency component of the disturbance.

V. Conclusions

This paper has shown that a persistent disturbance exerted on flexible space structures can be effectively accommodated employing the control design methodologies presented in this paper. Although the proposed methods exploit many well-known concepts and techniques, including the internal model principle and state-space control approaches, some significant contributions of the paper include 1) the concept of a disturbance rejection filter dipole and its use in classical control design, and 2) demonstration of the simplicity of the design methods on a generic example for a class of noncollocated control problems. It was also shown that both classical and state-space methods resort to non-minimum-phase filtering of unstably interacting modes. All of these methods require, nevertheless, a certain amount of trial and error.

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